## Design of Tee Reinforced Beam

Usually, the design problem of T- beam can be considered as a design of a section with pre - specified dimensions ( $h_{f}, b, b_{w}$, and $h$ ). Usually these dimensions have been determined as follows:

- $h_{f}$ and $b$ are both determined from slab design .
- $b_{w}$ and $h$ are determined based on one of following criteria:
- Based on architectural requirements.
- Based on shear requirements
- Based on strength.

Therefore, the main unknown of design problem is to determine the required reinforcement (As) and its details.

## Design Procedure

1. Compute required factored applied moment Mu based on given load (dead and live loads).
And calculate $\mathrm{M}_{\mathrm{n}}=\frac{\mathrm{M}_{\mathrm{u}}}{\varnothing} \quad$ and $\varnothing=0.9$ and will be checked later.

- In calculating Mu , the slab weight has been already included in the applied dead load, therefore only self-weight of beam stem should be added.

Slab self-weight has been included in the applied loads


Self-weight of beam must be added to applied loads

- Based on slab and beam data, determine the effective flange width "b" and as was discussed in previous article.

2. Compute the effective depth (d)
$\mathrm{d}_{\text {for one layer }}=\mathrm{h}-$ cover - stirrups $-\frac{\text { bar diameter }}{2}$
$\mathrm{d}_{\text {for two layer }}=\mathrm{h}-$ cover-stirrups - bar diameter $-\frac{\text { spacing between layers }}{2}$
3. Check if this section can be design with compression block in section flange or extend to section web based on following comparison: If

$\mathrm{M}_{\mathrm{n}} \leq 0.85 \mathrm{fc}^{\prime} \mathrm{h}_{\mathrm{f}} \mathrm{b}\left(\mathrm{d}-\frac{\mathrm{h}_{\mathrm{f}}}{2}\right) \quad$ then $\mathrm{a} \leq \mathrm{hf}$ Go to step 4.1

4.1 Design of a section with a $\leq \mathrm{hf}$

This section can be designed as a rectangular section with dimensions of $b$ and d.
$\rho_{\text {required }}=\frac{1}{\mathrm{~m}}\left(1-\sqrt{1-\frac{2 \mathrm{Rm}}{\mathrm{fy}}}\right)$
Where:
$\mathrm{m}=\frac{\mathrm{fy}}{0.85 \mathrm{fc}} \quad, \quad \mathrm{R}=\frac{\mathrm{M}_{\mathrm{u}} \times 10^{6}}{\emptyset \mathrm{bd}^{2}} \quad$ and $\emptyset=0.9$
As ${ }_{\text {Required }}=\rho_{\text {required }} \mathrm{bd}$
4.2 Design of a section with a $>\mathrm{hf}$
i. Compute the nominal moment that can be supported by flange overhangs.
$\mathrm{M}_{\mathrm{n} 1}=0.85 \mathrm{fc}^{\prime} \mathrm{h}_{\mathrm{f}}\left(\mathrm{b}-\mathrm{b}_{\mathrm{w}}\right)\left(\mathrm{d}-\frac{h_{f}}{2}\right)$
$\mathrm{A}_{\mathrm{sf}}=\frac{0.85 \mathrm{fc}^{\prime} \mathrm{hf}\left(\mathrm{b}-b_{w}\right)}{f_{y}}$
ii. Compute the remaining nominal strength that must be supported by section web:
$\mathrm{M}_{\mathrm{n} 2}=\mathrm{M}_{\mathrm{n}}-\mathrm{M}_{\mathrm{n} 1}$

For this moment $\mathrm{M}_{\mathrm{n} 2}$, the section can be designed a rectangular section with dimensions of $b_{w}$ and $d$ :
$\rho_{\text {required }}=\frac{1}{\mathrm{~m}}\left(1-\sqrt{1-\frac{2 \mathrm{Rm}}{\mathrm{fy}}}\right)$
Where:

$$
\begin{aligned}
& \mathrm{m}=\frac{\mathrm{fy}}{0.85 \mathrm{fc}} \quad, \quad \mathrm{R}=\frac{\mathrm{M}_{\mathrm{n} 2} \times 10^{6}}{b_{w} \mathrm{~d}^{2}} \\
& \mathrm{~A}_{\mathrm{s} 2}=\rho_{\text {required }} \mathrm{b}_{\mathrm{w}} \mathrm{~d} \\
& \mathrm{As}_{\text {required total }}=\mathrm{A}_{\mathrm{sf}}+\mathrm{A}_{\mathrm{s} 2}
\end{aligned}
$$

No. of bars $=\frac{A_{s} \text { required }}{A_{\text {bar }}}$ and check $\mathbf{b}_{\text {required }}$ with $\mathbf{b}$ provided
5. Check the $\mathrm{A}_{\mathrm{s} \text { provided }}$ with the maximum steel area permitted by ACI Code:
$\mathrm{A}_{\mathrm{s} \text { minimum }}=\frac{0.25 \sqrt{f c^{c}}}{f y} b_{w} d \geq \frac{1.4}{f y} b_{w} d \quad$ (Choose larger)
If $\mathrm{A}_{\mathrm{s} \text { required }}>\mathrm{A}_{\mathrm{s} \text { minimum }}$ O.k
If else:
Use $A_{s \text { provided }}=A_{s \text { minimum }}$
6. Check the $\mathrm{A}_{\text {s provided }}$ with the maximum steel area permitted by ACI code
$\rho_{\mathrm{w}}=\frac{A s}{b_{w} d}$
$\rho_{\mathrm{w} \text { max }}=0.85 \beta_{1} \frac{f c^{`}}{f y} \frac{\epsilon u}{\epsilon u+0.004}+\frac{A_{s f}}{b_{w} d}$
If $\rho_{\mathrm{w}} \leq \rho_{\mathrm{w} \text { max }}$ O.K
If else: $\rho_{\mathrm{w}}>\rho_{\mathrm{w} \text { max }}$

- Then the designer must increase one or more of beam dimensions. In practice m compression reinforcement is not used in T sections.

7. Check the assumption of $\varnothing=0.9$

Compute "a"
If $\mathrm{a} \leq \mathrm{h}_{\mathrm{f}}$
$\mathrm{a}=\frac{\mathrm{A}_{\mathrm{s}} \mathrm{fy}}{0.85 f \mathrm{c}^{\prime} \mathrm{b}}$
If else: $\mathrm{a}>\mathrm{h}_{\mathrm{f}}$
$\mathrm{a}=\frac{\left(\mathrm{A}_{\mathrm{s}}-\mathrm{A}_{\mathrm{sf}}\right) \mathrm{fy}}{0.85 \mathrm{fc} b_{w}}$
Then compute "c":
$\mathrm{c}=\frac{\mathrm{a}}{\beta 1}$
$\epsilon_{\mathrm{t}}=\frac{d t-c}{\mathrm{c}} \epsilon_{\mathrm{u}}$ where: $\epsilon_{u}=0.003$

- If $\epsilon_{\mathrm{t}} \geq 0.005$, then $\emptyset=0.9$
- If $\epsilon_{\mathrm{t}}<0.005$ then

$$
\emptyset=0.483+83.3 \times \epsilon_{\mathrm{t}}
$$

And retain to step of computing $\mathrm{M}_{\mathrm{n}}$
8. Draw final detail section

Example 1: Design a T -beam having cross section shown below according to ACI code -14 requirement. The beam is simply supported on span of (7.3152) m .the beam is subjected to service dead load (40.78) kN/m (without self-weight), and service live load (10) kN/m, Assume that the effective flange width has been calculated:


Assume the designer intends to use:

- $\mathrm{fc}^{`}=21 \mathrm{MPa}$
- $\mathrm{f}_{\mathrm{y}}=414 \mathrm{MPa}$
- For longitudinal reinforcement use $\emptyset 35 \mathrm{~mm}$
- One layer of reinforcement


## Solution:

1. Compute required factored applied moment Mu based on given load (dead and live loads).
$\mathrm{W}_{\text {self-weight }}=(0.55-0.09) \times 0.3 \times 24=3.312 \mathrm{kN} / \mathrm{m}$
$\mathrm{W}_{\mathrm{D}}=3.312+40.78=44.1 \mathrm{kN} / \mathrm{m}$
$\mathrm{Wu}=1.2 \mathrm{WD}+1.6 \mathrm{WL}=1.2 \times 44.1+1.6 \times 10=68.92 \mathrm{kN} / \mathrm{m}$
$\mathrm{Mu}=\frac{W_{u} \ell^{2}}{8}=\frac{68.92 \times 7.3152^{2}}{8}=461 \mathrm{kN} . \mathrm{m}$
$\mathrm{M}_{\mathrm{n}}=\frac{M_{u}}{\emptyset}=\frac{461}{0.9}=512 \mathrm{kN} . \mathrm{m}$
2. Compute the effective depth (d)
$\mathrm{d}_{\text {for one layer }}=\mathrm{h}-$ cover - stirrups $-\frac{\text { bar diameter }}{2}$
$\mathrm{d}_{\text {for one layer }}=550-40-10-\frac{35}{2}=482.5 \mathrm{~mm}$
3. Check if this section can be design with compression block in section flange or extend to section web based on following comparison
$\mathrm{M}_{\mathrm{n}}=512 \mathrm{kN} . \mathrm{m}$
$\mathrm{M}_{\mathrm{n} \text { flange }}=0.85 \mathrm{fc}^{\prime} \mathrm{h}_{\mathrm{f}} \mathrm{b}\left(\mathrm{d}-\frac{\mathrm{h}_{\mathrm{f}}}{2}\right)$
$\mathrm{M}_{\mathrm{n} \text { flange }}=0.85 \times 21 \times 90 \times 680 \times\left(482.5-\frac{90}{2}\right) \times 10^{-6}=478 \mathrm{kN} . \mathrm{m}$
$\mathrm{M}_{\mathrm{n}}>\mathrm{M}_{\mathrm{n} \text { flange }}$
$\therefore$ Design a section with a $>\mathrm{h}_{\mathrm{f}} \quad$ go to step 4.2

4.2 Design of a section with $\mathrm{a}>\mathrm{h}_{\mathrm{f}}$
i. Compute the nominal moment that can be supported by flange overhangs.

$$
\begin{aligned}
& M_{n 1}=0.85 f f^{`} h_{f}\left(b-b_{w}\right)\left(d-\frac{h_{f}}{2}\right) \\
& M_{n 1}=0.85 \times 21 \times 90 \times(680-300)\left(482.5-\frac{90}{2}\right) \times 10^{-6}=267 \mathrm{kN} . \mathrm{m}
\end{aligned}
$$

Steel reinforcement for this part will be:
$\mathrm{A}_{\mathrm{sf}}=\frac{0.85 \mathrm{fc} \mathrm{hf}\left(\mathrm{b}-b_{w}\right)}{f_{y}}=\frac{0.85 \times 21 \times 90 \times(680-300)}{414}=1474.6 \mathrm{~mm}^{2}$
ii. Compute the remaining nominal strength that must be supported by section web:
$\mathrm{M}_{\mathrm{n} 2}=\mathrm{M}_{\mathrm{n}}-\mathrm{M}_{\mathrm{n} 1}$
$\mathrm{M}_{\mathrm{n} 2}=512-268=244 \mathrm{kN} . \mathrm{m}$
For this moment $\mathrm{M}_{\mathrm{n} 2}$, the section can be designed a rectangular section with dimensions of $b_{w}$ and $d$ :
$\rho_{\text {required }}=\frac{1}{\mathrm{~m}}\left(1-\sqrt{1-\frac{2 \mathrm{Rm}}{\mathrm{fy}}}\right)$
$\mathrm{m}=\frac{\mathrm{fy}}{0.85 \mathrm{fc}}=\frac{414}{0.85 \times 21}=23.19, \quad \mathrm{R}=\frac{\mathrm{M}_{\mathrm{n} 2} \times 10^{6}}{b_{w} \mathrm{~d}^{2}}=\frac{244 \times 10^{6}}{300 \times 482.5^{2}}=3.49$
$\rho_{\text {required }}=\frac{1}{23.19}\left(1-\sqrt{1-\frac{2 \times 3.49 \times 23.19}{414}}\right)=9.47 \times 10^{-3}$

$$
\begin{aligned}
& \mathrm{A}_{\mathrm{s} 2}=\rho_{\mathrm{required}} \mathrm{~b}_{\mathrm{w}} \mathrm{~d}=9.47 \times 10^{-3} \times 300 \times 482.5=1371 \mathrm{~mm}^{2} \\
& \mathrm{~A}_{\mathrm{s} \text { required }}=\mathrm{A}_{\mathrm{sf}}+\mathrm{A}_{\mathrm{s} 2}=1474.6+1371=2845.6 \mathrm{~mm}^{2}
\end{aligned}
$$

No. of bars $=\frac{\mathrm{A}_{\mathrm{s}}}{\mathrm{A}_{\mathrm{b}}}=\frac{2845.6}{\frac{\pi}{4} \times 35^{2}}=\frac{2845.6}{962.11}=2.95 \approx 3$
Try $3 \emptyset 35 \mathrm{~mm}$

$$
\begin{aligned}
& \text { As provided }=3 \times 962.11=2886.33 \mathrm{~mm}^{2} \\
& \text { Check } b_{\text {required }}=40 \times 2+10 \times 2+3 \times 35+2 \times 35 \\
& b_{\text {required }}=275 \mathrm{~mm}<300 \mathrm{~mm} \mathrm{O.k}
\end{aligned}
$$

5. Check the $\mathrm{A}_{\mathrm{s} \text { provided }}$ with the maximum steel area permitted by ACI Code:
$\mathrm{A}_{\mathrm{s} \text { minimum }}=\frac{0.25 \sqrt{f c^{c}}}{f y} b_{w} d \geq \frac{1.4}{f y} b_{w} d \quad$ (Choose larger)
$\mathrm{A}_{\mathrm{s} \text { minimum }}=\frac{1.4}{f y} b_{w} d=\frac{1.4}{414} \times 300 \times 482.5=489.5 \mathrm{~mm}^{2}<\mathrm{A}_{\text {s provided }}$
6. Check the $\mathrm{A}_{\text {s provided }}$ with the maximum steel area permitted by ACI code
$\rho_{\mathrm{w}}=\frac{A s}{b_{w} d}=\frac{2886.33}{300 \times 482.5}=19.94 \times 10^{-3}$
$\rho_{\mathrm{w} \text { max }}=0.85 \beta_{1} \frac{f c^{`}}{f y} \frac{\epsilon u}{\epsilon u+0.004}+\frac{A_{s f}}{b_{w} d}$
$\rho_{\mathrm{w} \max }=0.85 \times 0.85 \times \frac{21}{414} \times \frac{0.003}{0.003+0.004}+\frac{1474.6}{300 \times 482.5}=25.89 \times 10^{-3}$
$\rho_{\mathrm{w}} \leq \rho_{\mathrm{w} \text { max }}$ O.K
7. Check the assumption of $\varnothing=0.9$
$\mathrm{a}>\mathrm{h}_{\mathrm{f}}$
$\mathrm{a}=\frac{(2886.33-1474.6) \times 414}{0.85 \times 21 \times 300}=109.14 \mathrm{~mm}$
Then compute " $c$ ":
$\mathrm{c}=\frac{\mathrm{a}}{\beta 1}=\frac{109.14}{0.85}=128.4 \mathrm{~mm}$
$\epsilon_{\mathrm{t}}=\frac{\mathrm{dt}-\mathrm{c}}{\mathrm{c}} \epsilon_{\mathrm{u}}=\frac{482.5-128.4}{128.4} \times 0.003=8.27 \times 10^{-3}>0.005$
$\therefore \emptyset=0.9$ O.K
8. Draw final detail section


Re-Design the previous example by using ETABS 2016 and compare the results of design and analysis


Design Moment and Flexural Reinforcement for Moment, $\mathbf{M}_{\mathrm{u}}$

|  | Design <br> -Moment <br> $\mathbf{k N}-\mathbf{m}$ | Design <br> +Moment <br> $\mathbf{k N}-\mathbf{m}$ | -Moment <br> Rebar <br> $\mathbf{m m}^{\mathbf{2}}$ | +Moment <br> Rebar <br> $\mathbf{m m}^{\mathbf{2}}$ | Minimum <br> Rebar <br> $\mathbf{m m}^{\mathbf{2}}$ | Required <br> Rebar <br> $\mathbf{m m}^{\mathbf{2}}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Top (+2 Axis) | 0 |  | 0 | 0 | 0 | 0 |
| Bottom (-2 Axis) |  | 460.6 | 0 | 2851 | 482 | 2851 |


| Method of design | Hand Calculation | ETABS (2016) | Difference \% |
| :---: | :---: | :---: | :---: |
| Mu kN.m | $\mathbf{4 6 1}(\mathbf{k N . m})$ | $\mathbf{4 6 0 . 6}(\mathrm{kN} . \mathrm{m})$ | $\mathbf{0 . 0 8} \%$ |
| Tension <br> Reinforcement $\left(\mathrm{mm}^{2}\right)$ | $\mathbf{2 8 4 5 . 6}\left(\mathrm{mm}^{2}\right)$ | $\mathbf{2 8 5 1}\left(\mathrm{mm}^{2}\right)$ | $\mathbf{0 . 1 9 \%}$ |

- In complex structure building when hand calculating is difficult to apply, it's useful to take one or two beams from that building and compare the results of analysis and design with results that obtained from structural software ( like ETABS in our course), this can be considered as verification for the program.

Example 2: Design the T-Beam shown below according to ACI-14 Code requirements. The floor slab is supported by 6.71 m simple span beams. Service loads are: $\mathrm{W}_{\mathrm{L}}=14.6 \mathrm{kN} / \mathrm{m}$ and $\mathrm{W}_{\text {dead }}=29.2 \mathrm{kN} / \mathrm{m}$


Assume the designer intend to use:

- $\mathrm{fc}^{`}=21 \mathrm{MPa}$ and $\mathrm{f}_{\mathrm{y}}=414 \mathrm{MPa}$
- Stirrups diameter is 10 mm
- $A_{b a r}=500 \mathrm{~mm}^{2}$ for $\emptyset 25 \mathrm{~mm}$
- Use one layer of reinforcement


## Solution:

1. Compute $\mathrm{M}_{\mathrm{u}}$

W self-weight $=(0.55-0.1) \times 0.3 \times 24=3.24 \mathrm{kN} / \mathrm{m}$
$\mathrm{W}_{\text {dead }}=29.2+3.24=32.4 \mathrm{kN} / \mathrm{m}$
$\mathrm{W}_{\mathrm{u}}=1.2 \mathrm{~W}_{\mathrm{D}}+1.6 \mathrm{~W}_{\mathrm{L}}=62.24 \mathrm{kN} / \mathrm{m}$
$\mathrm{Mu}=\frac{W_{u} \ell^{2}}{8}=\frac{62.24 \times 6.71^{2}}{8}=350 \mathrm{kN} . \mathrm{m}$

- Compute the nominal flexure moment $\mathrm{M}_{\mathrm{n}}$
$\mathrm{M}_{\mathrm{n}}=\frac{\mathrm{M}_{\mathrm{u}}}{\emptyset}==\frac{350}{0.9}=389 \mathrm{kN} . \mathrm{m}$
Compute the effective flange "b"
$\mathrm{b}=\mathrm{b}_{\mathrm{w}}+$ minimum $\left[\frac{S_{w}}{2}\right.$ or $8 h_{f}$ or $\left.\frac{e_{n}}{8}\right] \times 2$
$\mathrm{b}=300+\operatorname{minimum}\left[\frac{(2440-300)}{2}\right.$ or $8 \times 100$ or $\left.\frac{6710}{8}\right] \times 2$
$b=300+$ minimum [1070 or $\mathbf{8 0 0}$ or 839 ] $\times 2=300+800 \times 2=1900$

2. Compute the effective depth (d)
$\mathrm{d}_{\text {for one layer }}=\mathrm{h}-$ cover - stirrups $-\frac{\text { bar diameter }}{2}$
$\mathrm{d}_{\text {for one layer }}=550-40-10-\frac{25}{2}=487.5 \mathrm{~mm}$
3. Check if this section can be design with compression block in section flange or extend to section web based on following comparison
$\mathrm{M}_{\mathrm{n}}=389 \mathrm{kN} . \mathrm{m}$
$\mathrm{M}_{\mathrm{n} \text { flange }}=0.85 \mathrm{fc}^{\prime} \mathrm{h}_{\mathrm{f}} \mathrm{b}\left(\mathrm{d}-\frac{\mathrm{h}_{\mathrm{f}}}{2}\right)=0.85 \times 21 \times 100 \times 1900 \times\left(487.5-\frac{100}{2}\right)$
$\mathrm{M}_{\mathrm{n} \text { flange }}=1484 \mathrm{kN} . \mathrm{m}$
$\mathrm{M}_{\mathrm{n}}<\mathrm{M}_{\mathrm{n} \text { flange }}$
$\therefore$ Design a section with $\mathrm{a}<\mathrm{h}_{\mathrm{f}} \quad$ go to step 4.1

4.1 Design of a section with a $\leq \mathrm{hf}$

This section can be designed as a rectangular section with dimensions of $b$ and d.
$\rho_{\text {required }}=\frac{1}{\mathrm{~m}}\left(1-\sqrt{1-\frac{2 \mathrm{Rm}}{\mathrm{fy}}}\right)$
$\mathrm{m}=\frac{\mathrm{fy}}{0.85 \mathrm{fc}}=\frac{414}{0.85 \times 21}=23.19 \quad, \quad \mathrm{R}=\frac{350 \times 10^{6}}{0.9 \times 1900 \times 487.5^{2}}=0.861$
$\rho_{\text {required }}=\frac{1}{23.19}\left(1-\sqrt{1-\frac{2 \times 0.861 \times 23.19}{414}}\right)=2.13 \times 10^{-3}$
$\mathrm{As}_{\text {Required }}=\rho_{\text {required }}$ bd $=2.13 \times 10^{-3} \times 1900 \times 487.5=1973 \mathrm{~mm}^{2}$
No. of bars $=\frac{\mathrm{A}_{\mathrm{s}}}{\mathrm{A}_{\mathrm{b}}}=\frac{1973}{500}=\frac{1973}{500}=3.946 \approx 4$
Try $4 \emptyset 25 \mathrm{~mm}$
As provided $=4 \times 500=2000 \mathrm{~mm}^{2}$
Check $\mathrm{b}_{\text {required }}=40 \times 2+10 \times 2+4 \times 25+3 \times 25$
$\mathrm{b}_{\text {required }}=275 \mathrm{~mm}<300 \mathrm{~mm}$ O.k
5. Check the $\mathrm{A}_{\text {s provided }}$ with the maximum steel area permitted by ACI Code:

$$
\mathrm{A}_{\mathrm{s} \text { minimum }}=\frac{1.4}{f y} b_{w} d=\frac{1.4}{414} \times 300 \times 487.5=494.6 \mathrm{~mm}^{2}<\mathrm{A}_{\mathrm{s} \text { provided }} \mathrm{o} . \mathrm{k}
$$

6. Check the $\mathrm{A}_{\text {s provided }}$ with the maximum steel area permitted by ACI code

$$
\begin{aligned}
& \rho_{\mathrm{w}}=\frac{A s}{b_{w} d}=\frac{2000}{300 \times 487.5}=13.67 \times 10^{-3} \\
& \rho_{\mathrm{w} \max }=0.85 \beta_{1} \frac{f c^{\prime}}{f y} \frac{\epsilon u}{\epsilon u+0.004}+\frac{A_{s f}}{b_{w} d}
\end{aligned}
$$

$\mathrm{A}_{\mathrm{sf}}=\frac{0.85 \mathrm{fc}^{\prime} h_{f}\left(\mathrm{~b}-b_{w}\right)}{f_{y}}=\frac{0.85 \times 21 \times 100 \times(1900-300)}{414}=6899 \mathrm{~mm}^{2}$
$\rho_{\mathrm{w} \max }=0.85 \times 0.85 \times \frac{21}{414} \times \frac{0.003}{0.003+0.004}+\frac{6899}{300 \times 487.5}=62.88 \times 10^{-3}$
$\rho_{\mathrm{w}}<\rho_{\mathrm{w} \max }$ O.K
7. Check the assumption of $\varnothing=0.9$
$\mathrm{a} \leq \mathrm{h}_{\mathrm{f}}$
$\mathrm{a}=\frac{\mathrm{A}_{\mathrm{s}} \mathrm{fy}}{0.85 \mathrm{fc} \mathrm{b}}=\frac{2000 \times 414}{0.85 \times 21 \times 1900}=24.4 \mathrm{~mm}$
Then compute " $c$ ":
$\mathrm{c}=\frac{\mathrm{a}}{\beta 1}=\frac{24.4}{0.85}=28.7 \mathrm{~mm}$
$\epsilon_{\mathrm{t}}=\frac{d t-c}{\mathrm{c}} \epsilon_{\mathrm{u}}=\frac{487.5-28.7}{28.7} \times 0.003=0.0479>0.005$
$\therefore \quad \emptyset=0.90 . \mathrm{K}$
8. Draw final detail section


Example 3: Design a T- beam having a clear span of 6.0 m . a web thickness of 300 mm , and an overall depth of 645 mm . The beams spacing is 1.2 m center to center and the slab thickness is 100 mm . Design this beam for flexure to carries a total factored moment of $1300 \mathrm{~mm} \mathrm{kN} . \mathrm{m}$
Assume the designer intend to use:

- $\mathrm{fc}^{\prime}=28 \mathrm{MPa}$ and $\mathrm{fy}=400 \mathrm{MPa}$
- $\emptyset 32 \mathrm{~mm}$ for longitudinal reinforcement ( $\mathrm{A}_{\mathrm{bar}}=819 \mathrm{~mm}^{2}$ )
- $\quad 10 \mathrm{~mm}$ for stirrups
- Two layers of reinforcement.


## Solution:

1. Compute $\mathrm{M}_{\mathrm{u}}$
$\mathrm{M}_{\mathrm{u}}=1300 \mathrm{kN} . \mathrm{m}$
$\mathrm{M}_{\mathrm{n}}=\frac{M_{u}}{\varnothing}=\frac{1300}{0.9}=1444.44 \mathrm{kN} . \mathrm{m}$
2. Compute the effective depth (d)
$\mathrm{d}_{\text {for two layer }}=\mathrm{h}-$ cover-stirrups-bar diameter- $\frac{\text { spacing between layers }}{2}$
$\mathrm{d}_{\text {for two layer }}=645-40-10-32-\frac{25}{2}=550.5 \mathrm{~mm}$
$\mathrm{b}=300+$ minimum $\left[\frac{1200-300}{2}\right.$ or $8 \times 100$ or $\left.\frac{6000}{8}\right] \times 2$
$\mathrm{b}=300+\operatorname{minimum}[450$ or 800 or 750$] \times 2=300+450 \times 2=1200 \mathrm{~mm}$
3. Check if this section can be design with compression block in section flange or extend to section web based on following comparison $\mathrm{M}_{\mathrm{n}}=1444.44 \mathrm{kN} . \mathrm{m}$
$M_{n \text { flange }}=0.85 \mathrm{fc}^{\prime} \mathrm{h}_{\mathrm{f}} \mathrm{b}\left(\mathrm{d}-\frac{\mathrm{h}_{\mathrm{f}}}{2}\right)$
$\mathrm{Mn}_{\text {flange }}=0.85 \times 28 \times 100 \times 1200 \times\left(550.5-\frac{100}{2}\right) \times 10^{-6}=1429 \mathrm{kN} . \mathrm{m}$
$\mathrm{M}_{\mathrm{n}}>\mathrm{M}_{\mathrm{n} \text { flange }}$
$\therefore$ Design a section with $\mathrm{a}>\mathrm{h}_{\mathrm{f}} \quad$ go to step 4.2

4.2 Design of a section with a $>\mathrm{hf}$
i. Compute the nominal moment that can be supported by flange overhangs.
$\mathrm{M}_{\mathrm{n} 1}=0.85 \mathrm{fc}^{\prime} \mathrm{h}_{\mathrm{f}}\left(\mathrm{b}-\mathrm{b}_{\mathrm{w}}\right)\left(\mathrm{d}-\frac{h_{f}}{2}\right)$
$\mathrm{M}_{\mathrm{n} 1}=0.85 \times 28 \times 100 \times(1200-300)\left(550.5-\frac{100}{2}\right) \times 10^{-6}$
$\mathrm{M}_{\mathrm{n} 1}=1072 \mathrm{kN} . \mathrm{m}$
Steel reinforcement for this part will be:
$\mathrm{A}_{\mathrm{sf}}=\frac{0.85 \mathrm{fc}^{\circ} \mathrm{hf}\left(\mathrm{b}-b_{w}\right)}{f_{y}}=\frac{0.85 \times 28 \times 100 \times(1200-300)}{400}=5355 \mathrm{~mm}^{2}$
ii. Compute the remaining nominal strength that must be supported by section web:

$$
\begin{aligned}
& \mathrm{M}_{\mathrm{n} 2}=\mathrm{M}_{\mathrm{n}}-\mathrm{M}_{\mathrm{n} 1} \\
& \mathrm{M}_{\mathrm{n} 2}=1444.44-1072 \\
& \mathrm{M}_{\mathrm{n} 2}=372.44 \mathrm{kN} . \mathrm{m}
\end{aligned}
$$

For this moment $\mathrm{M}_{\mathrm{n} 2}$, the section can be designed a rectangular section with dimensions of $b_{w}$ and $d$ :
$\rho_{\text {required }}=\frac{1}{\mathrm{~m}}\left(1-\sqrt{1-\frac{2 \mathrm{Rm}}{\mathrm{fy}}}\right)$
$\mathrm{m}=\frac{\mathrm{fy}}{0.85 \mathrm{fc}}{ }^{\text {c }}=\frac{400}{0.85 \times 28}=16.81, \quad \mathrm{R}=\frac{\mathrm{M}_{\mathrm{n} 2} \times 10^{6}}{b_{w} \mathrm{~d}^{2}}=\frac{372.44 \times 10^{6}}{300 \times 550.5^{2}}=4.096$
$\rho_{\text {required }}=\frac{1}{16.81}\left(1-\sqrt{1-\frac{2 \times 4.096 \times 16.81}{400}}\right)=11.36 \times 10^{-3}$
$\mathrm{A}_{\mathrm{s} 2}=\rho_{\text {required }} \mathrm{b}_{\mathrm{w}} \mathrm{d}=11.36 \times 10^{-3} \times 300 \times 550.5=1876 \mathrm{~mm}^{2}$
$\mathrm{A}_{\mathrm{s} \text { required }}=\mathrm{A}_{\mathrm{sf}}+\mathrm{A}_{\mathrm{s} 2}=1876+5355=7231 \mathrm{~mm}^{2}$
No. of bars $=\frac{A_{s}}{A_{\text {bar }}}=\frac{7231}{819}=8.8 \approx 9$
Try $9 \varnothing 32 \mathrm{~mm}$
As provided $=9 \times 819=7371 \mathrm{~mm}^{2}$
Check $\mathrm{b}_{\text {required }}=40 \times 2+10 \times 2+5 \times 32+4 \times 32$
$\mathrm{b}_{\text {required }}=388 \mathrm{~mm}>300 \mathrm{~mm}$ Not O.K. use bundle bar
5. Check the $\mathrm{A}_{\text {s provided }}$ with the maximum steel area permitted by ACI Code:
$\mathrm{A}_{\mathrm{s} \text { minimum }}=\frac{0.25 \sqrt{f c^{c}}}{f y} b_{w} d \geq \frac{1.4}{f y} b_{w} d \quad$ (Choose larger)
$\mathrm{A}_{\mathrm{s} \text { minimum }}=\frac{1.4}{f y} b_{w} d=\frac{1.4}{414} \times 300 \times 550.5=578 \mathrm{~mm}^{2}<\mathrm{A}_{\text {s provided }} \quad$ O.K
6. Check the $\mathrm{A}_{\text {s provided }}$ with the maximum steel area permitted by ACI code
$\rho_{\mathrm{w}}=\frac{A s}{b_{w} d}=\frac{7371}{300 \times 550.5}=44.63 \times 10^{-3}$
$\rho_{\mathrm{w} \text { max }}=0.85 \beta_{1} \frac{f c^{\prime}}{f y} \frac{\epsilon u}{\epsilon u+0.004}+\frac{A_{s f}}{b_{w} d}$
$\rho_{\mathrm{w} \max }=0.85 \times 0.85 \times \frac{28}{400} \times \frac{0.003}{0.003+0.004}+\frac{5355}{300 \times 550.5}=54.1 \times 10^{-3}$
$\rho_{\mathrm{w}} \leq \rho_{\mathrm{w} \text { max }}$ O.K
7. Check the assumption of $\emptyset=0.9$
$\mathrm{a}>\mathrm{h}_{\mathrm{f}}$
$\mathrm{a}=\frac{(7231-5355) \times 400}{0.85 \times 28 \times 300}=105.1 \mathrm{~mm}$
Then compute " $c$ ":
$\mathrm{c}=\frac{\mathrm{a}}{\beta 1}=\frac{105.1}{0.85}=123.6 \mathrm{~mm}$
$\epsilon_{\mathrm{t}}=\frac{\mathrm{dt}-\mathrm{c}}{\mathrm{c}} \epsilon_{\mathrm{u}}=\frac{579-123.6}{123.6} \times 0.003=0.01105 \times 10^{-3}>0.005$
$\therefore \emptyset=0.9$ O.K
8. Draw final detail section
$2 \emptyset 12 \mathrm{~mm}$ nominal bars to

$9 \emptyset 32 \mathrm{~mm}$

Example 4: In slab-beam floor system, the flange width determined and as shown below. Design a T-section to resist moment of $1101 \mathrm{kN} . \mathrm{m}$ Assume the designer intend to use:

- $\mathrm{fc}=21 \mathrm{MPa}$
- $\mathrm{f}_{\mathrm{y}}=420 \mathrm{MPa}$
- $\emptyset 35 \mathrm{~mm}$ for longitudinal reinforcement $\left(\mathrm{A}_{\mathrm{bar}}=1000 \mathrm{~mm}^{2}\right)$
- $\varnothing 10 \mathrm{~mm}$ for stirrups
- Two layers of reinforcement.



## Solution:

1. Compute $\mathrm{M}_{\mathrm{u}}$
$\mathrm{M}_{\mathrm{u}}=1101 \mathrm{kN} . \mathrm{m}$
$\mathrm{M}_{\mathrm{n}}=\frac{M_{u}}{\varnothing}=\frac{1101}{0.9}=1223 \mathrm{kN} . \mathrm{m}$
2. Compute the effective depth (d)
$\mathrm{d}_{\text {for two layer }}=\mathrm{h}-$ cover-stirrups-bar diameter- $\frac{\text { spacing between layers }}{2}$
$\mathrm{d}_{\text {for two layer }}=660-40-10-35-\frac{25}{2}=562.5 \mathrm{~mm}$
$\mathrm{b}=1200 \mathrm{~mm}$
3. Check if this section can be design with compression block in section flange or extend to section web based on following comparison
$\mathrm{M}_{\mathrm{n}}=1223 \mathrm{kN} . \mathrm{m}$
$\mathrm{M}_{\mathrm{n} \text { flange }}=0.85 \mathrm{fc}^{\prime} \mathrm{h}_{\mathrm{f}} \mathrm{b}\left(\mathrm{d}-\frac{\mathrm{h}_{\mathrm{f}}}{2}\right)$
$\mathrm{Mn}_{\text {flange }}=0.85 \times 21 \times 100 \times 1200 \times\left(562.5-\frac{100}{2}\right) \times 10^{-6}=1097.8 \mathrm{kN} . \mathrm{m}$
$M_{n}>M_{n}$ flange
$\therefore$ Design a section with $\mathrm{a}>\mathrm{h}_{\mathrm{f}}$
go to step 4.2

4.2 Design of a section with a $>\mathrm{hf}$
i. Compute the nominal moment that can be supported by flange overhangs.
$\mathrm{M}_{\mathrm{n} 1}=0.85 \mathrm{fc}^{\prime} \mathrm{h}_{\mathrm{f}}\left(\mathrm{b}-\mathrm{b}_{\mathrm{w}}\right)\left(\mathrm{d}-\frac{h_{f}}{2}\right)$
$\mathrm{M}_{\mathrm{n} 1}=0.85 \times 21 \times 100 \times(1200-300)\left(562.5-\frac{100}{2}\right) \times 10^{-6}$
$\mathrm{M}_{\mathrm{n} 1}=823.23 \mathrm{kN} . \mathrm{m}$
Steel reinforcement for this part will be:
$\mathrm{A}_{\mathrm{sf}}=\frac{0.85 \mathrm{fc} \mathrm{hf}\left(\mathrm{b}-b_{w}\right)}{f_{y}}=\frac{0.85 \times 21 \times 100 \times(1200-300)}{420}=3825 \mathrm{~mm}^{2}$
ii. Compute the remaining nominal strength that must be supported by section web:

$$
\begin{aligned}
& \mathrm{M}_{\mathrm{n} 2}=\mathrm{M}_{\mathrm{n}}-\mathrm{M}_{\mathrm{n} 1} \\
& \mathrm{M}_{\mathrm{n} 2}=1223-823.32 \\
& \mathrm{M}_{\mathrm{n} 2}=400 \mathrm{kN} . \mathrm{m}
\end{aligned}
$$

For this moment $\mathrm{M}_{\mathrm{n} 2}$, the section can be designed a rectangular section with dimensions of $b_{w}$ and d:
$\rho_{\text {required }}=\frac{1}{\mathrm{~m}}\left(1-\sqrt{1-\frac{2 \mathrm{Rm}}{\mathrm{fy}}}\right)$
$\mathrm{m}=\frac{\mathrm{fy}}{0.85 \mathrm{fc}}=\frac{420}{0.85 \times 21}=23.53, \quad \mathrm{R}=\frac{\mathrm{M}_{\mathrm{n} 2} \times 10^{6}}{b_{w} \mathrm{~d}^{2}}=\frac{400 \times 10^{6}}{300 \times 562.5^{2}}=4.21$
$\rho_{\text {required }}=\frac{1}{23.53}\left(1-\sqrt{1-\frac{2 \times 4.21 \times 23.53}{420}}\right)=11.6 \times 10^{-3}$
$\mathrm{A}_{\mathrm{s} 2}=\rho_{\text {required }} \mathrm{b}_{\mathrm{w}} \mathrm{d}=11.6 \times 10^{-3} \times 300 \times 562.5=1958 \mathrm{~mm}^{2}$
$\mathrm{A}_{\mathrm{s} \text { required }}=\mathrm{A}_{\mathrm{sf}}+\mathrm{A}_{\mathrm{s} 2}=3825+1958=5783 \mathrm{~mm}^{2}$
No. of bars $=\frac{A_{s}}{A_{\text {bar }}}=\frac{5783}{1000}=5.8 \approx 6$
Try $6 \emptyset 35 \mathrm{~mm}$

As provided $=6 \times 1000=6000 \mathrm{~mm}^{2}$
Check $\mathrm{b}_{\text {required }}=40 \times 2+10 \times 2+3 \times 35+2 \times 35$
$b_{\text {required }}=275 \mathrm{~mm}<300 \mathrm{~mm}$ O.K.
5. Check the $\mathrm{A}_{\mathrm{s} \text { provided }}$ with the maximum steel area permitted by ACI Code:
$\mathrm{A}_{\mathrm{s} \text { minimum }}=\frac{0.25 \sqrt{f c^{c}}}{f y} b_{w} d \geq \frac{1.4}{f y} b_{w} d \quad$ (Choose larger)
$\mathrm{A}_{\mathrm{s} \text { minimum }}=\frac{1.4}{f y} b_{w} d=\frac{1.4}{420} \times 300 \times 562.5=562.5 \mathrm{~mm}^{2}<\mathrm{A}_{\mathrm{s} \text { provided }}$ O.K
6. Check the $\mathrm{A}_{\text {s provided }}$ with the maximum steel area permitted by ACI code
$\rho_{\mathrm{w}}=\frac{A s}{b_{w} d}=\frac{6000}{300 \times 562.5}=35.56 \times 10^{-3}$
$\rho_{\mathrm{w} \text { max }}=0.85 \beta_{1} \frac{f c^{\prime}}{f y} \frac{\epsilon u}{\epsilon u+0.004}+\frac{A_{s f}}{b_{w} d}$
$\rho_{\mathrm{w} \text { max }}=0.85 \times 0.85 \times \frac{21}{420} \times \frac{0.003}{0.003+0.004}+\frac{3825}{300 \times 562.5}=36.37 \times 10^{-3}$
$\rho_{\mathrm{w}} \leq \rho_{\mathrm{w} \max }$ O.K
7. Check the assumption of $\emptyset=0.9$
$\mathrm{a}>\mathrm{h}_{\mathrm{f}}$
$\mathrm{a}=\frac{\left(\mathrm{A}_{\mathrm{s}}-\mathrm{A}_{\text {sf }}\right) \mathrm{fy}}{0.85 \mathrm{fc} b_{w}}$
$\mathrm{a}=\frac{(6000-3825) \times 420}{0.85 \times 21 \times 300}=170.5 \mathrm{~mm}$
Then compute " $c$ ":
$\mathrm{c}=\frac{\mathrm{a}}{\beta 1}=\frac{170.5}{0.85}=200.6 \mathrm{~mm}$
$\epsilon_{\mathrm{t}}=\frac{\mathrm{dt}-\mathrm{c}}{\mathrm{c}} \epsilon_{\mathrm{u}}=\frac{592.5-200.6}{200.6} \times 0.003=5.86 \times 10^{-3}>0.005$
$\therefore \emptyset=0.9$ O.K
8. Draw final detail section $2 \emptyset 12 \mathrm{~mm}$ nominal bars to


